Physics III
ISI B.Math
End of Semester Exam December 07, 2011

Total Marks: 75
Time: 3 hours
Answer ANY 5

Question 1. Total Marks:3+6+6

a.) Show that the dipole term for electric potential $V_{dipole}(r) = (\frac{1}{4\pi\epsilon_0 r^2}) \int r' \cos\theta' \rho(r') d\tau'$ can be written as $V_{dipole}(r) = \frac{1}{4\pi\epsilon_0} \frac{\overrightarrow{p}.\widehat{r}}{r^2}$ where $\overrightarrow{p} = \int \overrightarrow{r'} \rho(r') d\tau'$

b.) Show that the necessary and sufficient condition for $\overrightarrow{p}$ to be independent of the choice of origin is that the total charge is zero.

c.) There are four charges with their position coordinates as follows: $+3q, +q, -2q, -2q$ at $(0,0,a)$, $(0,0,-a)$, $(2a,-2a,0)$, and $(-2a,2a,0)$ respectively. Show that the potential at (r,θ,ϕ) for $r \gg a$ is $V(r) \simeq \frac{1}{2\pi\epsilon_0} \frac{qa\cos\theta}{r^2}$

Question 2. Total Marks:8+4+3

a.) Using the integral form of the equations $\nabla.D = \rho_f$ and ∇ x $E = -\frac{\partial B}{\partial t}$ derive the boundary conditions $D_1^{\perp} - D_1^{\perp} = \sigma_f$ and $E_1^{\parallel} - E_2^{\parallel} = 0$ at the boundary of two dielectric media.

b.) What form do these boundary conditions take in the special case of a linear dielectric material?

c.) Use the above to show that electric field lines bend at the boundary of linear dielectric media with no free charge at their boundary according to the rule $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}$ where θ_1 and θ_2 are the angles of the electric field line with the normal at the point of intersection

Question 3. Total Marks:8+7

a.) Starting with the formula for the electric potential due a single dipole $V_{dipole}(r) = \frac{1}{4\pi\epsilon_0} \frac{\overrightarrow{p}.\widehat{r}}{r^2}$ show that the total potential V due to a piece of polarized material with polarization $\overrightarrow{P}$ (which means that dipole density per unit volume is $\overrightarrow{P}$) can be written as $V = V_1 + V_2$ where V_1 is the potential coming from a "bound charge density" $\rho_b = -\nabla.\overrightarrow{P}$ and V_2 is the potential coming from a surface charge density given by $\sigma = \overrightarrow{P}.\hat{n}$ [Exact expressions of V_1 and V_2 required]

b.) Suppose a sphere of radius R carries a polarization $\overrightarrow{P} = k\overrightarrow{r}$. Find the Electric field inside and outside the sphere.

Question 4. Total Marks:6+4+5

a.) Show that any Magnetic field B can always be written as $B = \nabla$ x $\overrightarrow{A}$. Show that there are many $\overrightarrow{A}$ that can give rise to the same B. If $\overrightarrow{A}$ and $\overrightarrow{A'}$ produce the same magnetic field, what must be the relation between $\overrightarrow{A}$ and $\overrightarrow{A'}$? Show that we can always choose $\overrightarrow{A}$ such that $\nabla.\overrightarrow{A} = 0$

b.) Use the result of part a.) above and ∇ x $B = \mu_0 \overrightarrow{J}$ to show that $\overrightarrow{A}$ satisfies the equation $\nabla^2 \overrightarrow{A} = -\mu_0 \overrightarrow{J}$. Write down the solution of $\overrightarrow{A}$ in terms of $\overrightarrow{J}$

c.) Using the result in b.) derive Neumann formula which says that the flux of magnetic field Φ through one loop due to a current I running though another is given by $\Phi = \frac{\mu_0 I}{4\pi} \oint \oint \frac{d\overrightarrow{l_1}.d\overrightarrow{l_2}}{|r_1 - r_2|}$

Question 5. Total Marks:5+9+1

a.) Starting from the experimentally observed fact that the total amount of electric charge is conserved in any given volume, derive the continuity equation $\nabla.\overrightarrow{J} + \frac{\partial \rho}{\partial t} = 0$

b.) A finite spherical conductor of electrical conductivity σ has a unit charge density ρ_0 at time $t = 0$.

Show that as time goes by, the electric field and the charge density inside the conductor are given by $\rho = \rho_0 f(t)$ and $\overrightarrow{E}(\overrightarrow{r}, t) = (\frac{\rho_0}{3\epsilon_0})\overrightarrow{r} f(t)$ where $f(t) = e^{-\sigma/\epsilon_0}$

[Hint: Use $\overrightarrow{J} = \sigma \overrightarrow{E}$ relating $\overrightarrow{J}$ and $\overrightarrow{E}$, the continuity equation above, Maxwell's equation $\nabla.\overrightarrow{E} = \frac{\rho}{\epsilon_0}$ and spherical symmetry.

c.) What happens to the energy stored in the system?

Question 6. Total Marks: 4+4+7

a.) Write Maxwell's equations in a region of space where there are no charges and currents.

b.) Derive the wave equations for electric and magnetic fields in free space.

c.) Let the real parts of $\overrightarrow{E} = \overrightarrow{E_0} e^{i(kz-\omega t)}$ and $\overrightarrow{B} = \overrightarrow{B_0} e^{i(kz-\omega t)}$ be solutions of the above equations. What is the direction of the propagation of these wave solutions? Show that both $\overrightarrow{E_0}$ and $\overrightarrow{B_0}$ lie in the x,y plane and that $\overrightarrow{B_0} = \frac{k}{\omega}$ ($\hat{z}$ x $\overrightarrow{E_0}$)

Useful Formulae:

∇x(∇x$\overrightarrow{C}$)=∇($\nabla.\overrightarrow{C}$) - $\nabla^2 \overrightarrow{C}$,

$\nabla.(f\overrightarrow{C}) = f(\nabla.\overrightarrow{C}) + \overrightarrow{C}.\nabla(f)$,

$\nabla.\overrightarrow{C} = (\frac{1}{r^2})\frac{\partial}{\partial r}(r^2 C_r)+$ terms containing derivatives of θ and ϕ